ISRAEL JOURNAL OF MATHEMATICS, Vol. 71, No. 3, 1990

MEASURE-VALUED MARKOV BRANCHING PROCESSES CONDITIONED ON NON-EXTINCTION

BY

STEVEN N. EVANS[a],† AND EDWIN PERKINS[b],‡

[a]*Department of Statistics, University of California at Berkeley, 367 Evans Hall, Berkeley, CA 94720, USA;*
and [b]*Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Y4, Canada*

ABSTRACT

We consider a particular class of measure-valued Markov branching processes that are constructed as "superprocesses" over some underlying Markov process. Such a process X dies out almost surely, so we introduce various conditioning schemes which keep X alive at large times. Under suitable hypotheses, which include the convergence of the semigroup for the underlying process to some limiting probability measure ν, we show that the conditional distribution of $t^{-1}X_t$ converges to that of $Z\nu$ as $t \to \infty$, where Z is some strictly positive, real random variable.

1. Introduction and statement of results

Let E be a locally compact, second countable, Hausdorff topological space. Denote by $M(E)$ the space of finite measures on E equipped with the topology of weak convergence. Suppose that $\xi = (\Omega, \mathfrak{F}, \mathfrak{F}_t, \theta_t, \xi_t, P^x)$ is a Borel right Markov process in E with semigroup (P_t) such that $P_t 1 = 1$, $t \geq 0$.

From a more general construction in [5] (see, also, [2]) we have that for each bounded, non-negative Borel function f: $E \to \mathbb{R}$ the integral equation

$$v_t(x) = P_t f(x) - \int_0^t P_s(x, v_{t-s}^2) ds$$

has a unique solution $v_t = V_t f$, and there exists a unique Markov semigroup, (Q_t), on $M(E)$ for which

†Research supported in part by NSF grant DMS 8701212.

‡Research supported in part by an NSERC operating grant.

Received October 11, 1989

$$\int Q_t(\mu, d\nu)e^{-\nu(f)} = \exp(-\mu(V_t f)) \tag{1.1}$$

for all such f. Moreover, (Q_t) is the semigroup of an $M(E)$-valued right process $X = (W, \mathcal{G}, \mathcal{G}_t, \Theta_t, X_t, \mathbb{P}^\mu)$.

The first construction of this type appears in [12] for the case when (P_t) is a Feller semigroup. We refer the reader to the Introduction of [5] for a representative bibliographic selection from the extensive amount of work that recently has been done in studying various aspects of this and related classes of measure-valued process.

Observe from (1.1) that

$$\mathbb{P}^\mu[\exp(-\lambda X_t(1))] = \exp(-\mu(1)\lambda(1+\lambda t)^{-1}) \tag{1.2}$$

for $\lambda \geq 0$ and so

$$\mathbb{P}^\mu[X_t = 0] = \exp(-\mu(1)t^{-1}). \tag{1.3}$$

It is also clear from (1.1) that the null measure is a trap for X. Combining these two observations, we see that $X_t = 0$ for all t sufficiently large $\mathbb{P}^\mu$-a.s.

Our aim in this paper is to study the long-term behaviour of X_t on the rare event that $X_t \neq 0$. More precisely, we introduce various conditioning régimes that ensure that $X_t \neq 0$ and then obtain distributional limit theorems for $a_t X_t$ as $t \to \infty$, where (a_t) is some suitable family of constants. This type of result is familiar from the branching process literature (see, for example, Theorem 9.2 of [1]).

For $0 \leq T < \infty$ and $\mu \neq 0$ define

$$\mathbb{P}^\mu_T[\cdot] = \mathbb{P}^\mu[\cdot \mid X_s \neq 0,\ 0 \leq s \leq T].$$

From (1.3) we see that if $0 \leq t < T$ then $\mathbb{P}^\mu_{T|\mathcal{G}_t}$, the restriction of $\mathbb{P}^\mu_T$ to $\mathcal{G}_t$, is given by

$$\mathbb{P}^\mu_{T|\mathcal{G}_t}[\cdot] = \mathbb{P}^\mu\left[\cdot\left\{\frac{1-\exp(-X_t(1)(T-t)^{-1})}{1-\exp(-\mu(1)T^{-1})}\right\}\right]. \tag{1.4}$$

We also want to construct for $\mu \neq 0$ a probability $\mathbb{P}^\mu_\infty$ which has the intuitive interpretation of $\mathbb{P}^\mu[\cdot \mid X_s \neq 0,\ 0 \leq s < \infty]$. From (1.4) we find that if Φ is bounded and $\mathcal{G}_t$-measurable then

$$\lim_{T\to\infty} \mathbb{P}^\mu_T[\Phi] = \mu(1)^{-1}\,\mathbb{P}^\mu[\Phi X_t(1)].$$

Now Proposition 2.7 of [5] shows that the function $\nu \mapsto \nu(1)$ is invariant for (Q_t) (this also may be seen directly by differentiating both sides of (1.2) at $\lambda = 0$). Applying the observations on p. 298 of [11] we see that under suitable conditions on W (which are satisfied by the canonical set-up in [5], for example) there does indeed exist a probability measure $\mathbb{P}_\infty^\mu$ on W such that

$$\mathbb{P}_{\infty|\mathcal{G}_t}^\mu[\cdot] = \mu(1)^{-1}\mathbb{P}^\mu[\cdot X_t(1)]. \tag{1.5}$$

Moreover, under these conditions $(W,\mathcal{G},\mathcal{G}_t,\Theta_t,X_t,\mathbb{P}_\infty^\mu)$ is a conservative right process on the space $\{\nu \in M(E): \nu \neq 0\}$. In any case, for each $t \geq 0$ we may use the right-hand side of (1.5) to define a probability measure on $\mathcal{G}_t$. We may unambiguously refer to this measure as $\mathbb{P}_\infty^\mu$ because for $0 \leq s \leq t$ the measure constructed in the same manner but with t replaced by s coincides with the former measure restricted to $\mathcal{G}_s$. The measure $\mathbb{P}_\infty^\mu$ is constructed and characterized in [9] for the case when (P_t) is Feller.

Recall that we are trying to find constants (a_t) such that under appropriate conditioning $a_t X_t$ converges in distribution to some random finite measure Y as $t \to \infty$. To get a feel for the sort of conditions that will be required for this to happen, suppose that we have (a_t) such that $a_t X_t$ converges in distribution to some almost surely non-zero random measure Y under $\mathbb{P}_\infty^\mu$. Then, for each bounded, continuous function $f\colon E \to \mathbb{R}$, we must have $\lim_{t\to\infty}\mathbb{P}_\infty^\mu[X_t(f)/X_t(1)] = m(f)$, where m is the expectation measure for the random probability measure $Y(\cdot)/Y(1)$. Proposition 2.7 of [5] gives that

$$\begin{aligned}\mathbb{P}_\infty^\mu[X_t(f)/X_t(1)] &= \mu(1)^{-1}\mathbb{P}^\mu[X_t(f)] \\ &= \mu(1)^{-1}\mu P_t f.\end{aligned}$$

So, at the very least, we will require some form of ergodic behaviour for the semigroup (P_t). With this in mind, we record the following hypotheses.

Hypothesis (I). There exists a probability measure ν on E such that for each bounded, continuous function $f\colon E \to \mathbb{R}$ we have that $P_t f \to \nu(f)$ uniformly on compact subsets of E as $t \to \infty$.

Hypothesis (II). There exists a probability measure ν on E such that $P_t(x,\cdot) \ll \nu$ for all $t > 0$ and $x \in E$; and for each bounded, continuous function $f\colon E \to \mathbb{R}$ we have that $P_t f \to \nu(f)$ pointwise as $t \to \infty$.

We are now ready to state our main result.

Theorem. *Consider $\mu \in M(E)\setminus\{0\}$. Under Hypothesis (I) or Hypothesis (II) the following hold.*

(i) *For* $\beta \in [1,\infty[$ *the distribution of* $t^{-1}X_t$ *under* $\mathbf{P}^{\mu}_{\beta t}$ *converges weakly as* $t \to \infty$ *to that of the random measure* $Z_\beta \nu$, *where* Z_β *is a non-negative random variable having density*

$$x \mapsto \begin{cases} e^{-x}, & \beta = 1, \\ \beta[e^{-x} - e^{-\beta x/(\beta-1)}], & \beta > 1, \end{cases}$$

with respect to Lebesgue measure on $[0,\infty[$.

(ii) *The distribution of* $t^{-1}X_t$ *under* $\mathbf{P}^{\mu}_{\infty}$ *converges weakly as* $t \to \infty$ *to that of the random measure* $Z_\infty \nu$, *where* Z_∞ *is a non-negative random variable having density* $x \mapsto xe^{-x}$ *with respect to Lebesgue measure on* $[0,\infty[$.

We give the proof of the theorem in §2 and then give some examples in §3 of specific classes of processes which satisfy one or the other of the hypotheses.

The conclusion of the theorem should hold more generally. For instance, even if (P_t) has more than one invariant probability measure it should be true that the conclusion holds when ν is an extremal invariant probability measure and μP_t converges to ν. Unfortunately, we are unable to obtain a result of this generality. We do remark, however, that the only place where Hypotheses I or II are used is in Lemma 2.3 to ensure that

$$\lim_{t\to\infty} t^{-1} \int_0^t \mu P_s(P_{t-s}f - \nu(f))^2\, ds = 0,$$

and in particular special cases it may be possible to check this condition even when Hypothesis I or II is not satisfied.

We end this section with some remarks about the connection between our results and the so-called "cluster representation" of X_t. It is known that we may construct on some probability space independent, identically distributed, non-zero random measures $\{Y_1^{\mu,t}, Y_2^{\mu,t}, \ldots\}$ and an independent Poisson random variable $N(\mu,t)$ with expectation $\mu(1)t^{-1}$ such that under $\mathbf{P}^{\mu}$ the distribution of X_t is that of $\sum_{i=1}^{N(\mu,t)} Y_i^{\mu,t}$ (see, for example, Prop. III.1.1 of [3] or Lemma 1.4 of [4]). Therefore, the distribution of X_t under $\mathbf{P}^{\mu}_t$ is just the conditional distribution of $\sum_{i=1}^{N(\mu,t)} Y_i^{\mu,t}$ given $N(\mu,t) \neq 0$. An easy calculation shows that, conditional on $N(\mu,t) \neq 0$, the distribution of $N(\mu,t)$ converges weakly to the point mass at 1 as $t \to \infty$. We may thus conclude from part (i) of the Theorem with $\beta = 1$ that $t^{-1}Y_1^{\mu,t}$ converges in distribution to $Z_1\nu$ as $\to \infty$.

2. Proof of the Theorem

LEMMA 2.1. *Consider* $\mu \in M(E)\setminus\{0\}$.

(i) *For* $\beta \in [1,\infty[$ *the distribution of* $t^{-1}X_t(1)$ *under* $\mathbb{P}^\mu_{\beta t}$ *converges weakly as* $t \to \infty$ *to that of* Z_β.

(ii) *The distribution of* $t^{-1}X_t(1)$ *under* $\mathbb{P}^\mu_\infty$ *converges weakly as* $t \to \infty$ *to that of* Z_∞.

PROOF. (i) First consider the case $\beta = 1$. From (1.2) and (1.3) we see that for $\lambda \geq 0$

$$\mathbb{P}^\mu_t[\exp(-\lambda t^{-1}X_t(1))] = \frac{\exp(-\mu(1)\lambda(1+\lambda)^{-1}t^{-1}) - \exp(-\mu(1)t^{-1})}{1-\exp(-\mu(1)t^{-1})}$$

which converges to $(1+\lambda)^{-1}$ as $t \to \infty$. As $\lambda \mapsto (1+\lambda)^{-1}$ is the Laplace transform of Z_1 the result follows.

Now consider $\beta \in]1,\infty[$. From (1.4) and (1.2) we see that

$$\begin{aligned}
&\mathbb{P}^\mu_{\beta t}[\exp(-\lambda t^{-1}X_t(1))]\\
&= \mathbb{P}^\mu\left[\exp(-\lambda t^{-1}X_t(1))\left\{\frac{1-\exp(-X_t(1)(\beta-1)^{-1}t^{-1})}{1-\exp(-\mu(1)\beta^{-1}t^{-1})}\right\}\right]\\
&= \frac{\exp(-\mu(1)\lambda t^{-1}(1+\lambda)^{-1})}{1-\exp(-\mu(1)\beta^{-1}t^{-1})}\\
&\quad - \frac{\exp(-\mu(1)\{\lambda t^{-1}+(\beta-1)^{-1}t^{-1}\}\{1+\lambda+(\beta-1)^{-1}\}^{-1})}{1-\exp(-\mu(1)\beta^{-1}t^{-1})}\\
&\to \beta(\beta-1)^{-1}(1+\lambda)^{-1}\{1+\lambda+(\beta-1)^{-1}\}^{-1}
\end{aligned}$$

as $t \to \infty$, and this last expression is readily verified to be the Laplace transform of Z_β.

(ii) From the above we find that

$$\begin{aligned}
\mathbb{P}^\mu_\infty[\exp(-\lambda t^{-1}X_t(1))] &= \lim_{\beta\to\infty} \mathbb{P}^\mu_{\beta t}[\exp(-\lambda t^{-1}X_t(1))]\\
&= (1+\lambda)^{-2}\exp(-\mu(1)\lambda t^{-1}(1+\lambda)^{-1})\\
&\to (1+\lambda)^{-2}
\end{aligned}$$

as $t \to \infty$, and this last expression is of course the Laplace transform of Z_∞. □

LEMMA 2.2. *Consider $\mu \in M(E)\setminus\{0\}$. For $\beta \in\ ]1,\infty[$ and any measurable function $G\colon [0,\infty[\ \times M(E) \to [0,\infty[$ one has that*

$$\limsup_{t\to\infty} \mathbb{P}^{\mu}_{\beta t}[G(t,X_t)] \le (1-\beta^{-1})^{-1} \limsup_{t\to\infty} \mathbb{P}^{\mu}_{\infty}[G(t,X_t)].$$

PROOF. This is clear from (1.4), (1.5) and the inequality $1-e^{-x} \le x$, $x \ge 0$. □

LEMMA 2.3. *Consider $\mu \in M(E)\setminus\{0\}$. Suppose that either Hypothesis (I) or Hypothesis (II) holds. Then for each bounded, continuous function $f\colon E \to \mathbb{R}$,*

$$\limsup_{t\to\infty} t^{-1}\mathbb{P}^{\mu}[(X_t(f)-\nu(f)X_t(1))^2] = 0.$$

PROOF. Observe from Proposition 2.7 of [5] that

$$\begin{aligned}\mathbb{P}^{\mu}[(X_t(f)-\nu(f)X_t(1))^2] &= (\mu P_t f - \mu(1)\nu(f))^2 + 2\int_0^t \mu P_s(P_{t-s}f-\nu(f))^2\,ds \\ &= \gamma(t),\end{aligned}$$

say.

Assume firstly that Hypothesis (I) holds. Since μP_s converges weakly to $\mu(1)\nu$ as $s \to \infty$, for each $\epsilon > 0$ there exists $M \ge 0$ and a compact set K such that $\mu P_s(K) > \mu(1)(1-\epsilon)$ for all $s > M$. Thus

$$\begin{aligned}\limsup_{t\to\infty} \frac{1}{2} t^{-1}\gamma(t) &\le \limsup_{t\to\infty} \mu(1)(1-\epsilon)t^{-1}\int_M^t \sup_{x\in K}(P_{t-s}f(x)-\nu(f))^2\,ds \\ &\quad + \limsup_{t\to\infty} \mu(1)\epsilon t^{-1}\int_M^t (2\sup_{x\in E}|f(x)|)^2\,ds \\ &= 4\epsilon\mu(1)\sup_{x\in E}|f(x)|^2,\end{aligned}$$

and so $\limsup_{t\to\infty} t^{-1}\gamma(t) = 0$.

Assume now that Hypothesis (II) holds. As $\nu P_s = \nu$ for all $s \ge 0$ we see that

$$\limsup_{t\to\infty} t^{-1}\int_0^t \nu P_s(P_{t-s}f-\nu(f))^2\,ds = \limsup_{t\to\infty}\int_0^1 \nu([P_{t(1-s)}f-\nu(f)]^2)\,ds = 0, \tag{2.3.1}$$

by a change of variables and bounded convergence. Hence for each $\eta > 0$

$$\lim_{t\to\infty} t^{-1}\int_\eta^t P_{s-\eta}(P_{t-s}f-\nu(f))^2(\cdot)\,ds = 0 \tag{2.3.2}$$

in νP_η-measure (which is to say, in ν-measure). If $\limsup_{t\to\infty} t^{-1}\gamma(t) > 0$ then we can find $\delta > 0$, $\eta > 0$ and a sequence $\{t_n\}$ with $t_n \to \infty$ for which

$$t_n^{-1}\mu P_\eta \int_\eta^{t_n} P_{s-\eta}(P_{t-s}f - \nu(f))^2 ds > \delta, \qquad \forall n. \tag{2.3.3}$$

Now from (2.3.2) there exists a subsequence $\{u_n\} \subset \{t_n\}$ such that

$$\lim_{n\to\infty} u_n^{-1} \int_\eta^{u_n} P_{s-\eta}(P_{t-s}f - \nu(f))^2(\cdot)ds = 0$$

ν-a.e., and thus μP_η-a.e. also. Applying bounded convergence we obtain a contradiction to (2.3.3). □

We can now complete the proof of the Theorem. From arguments almost identical to those in Theorem 4.2 of [7], it suffices to show that for each bounded, continuous function f: $E \to \mathbb{R}$ we have that the distribution of $t^{-1}X_t(f)$ under $\mathbb{P}^\mu_{\beta t}$ (respectively, $\mathbb{P}^\mu_\infty$) converges weakly as $t \to \infty$ to that of $Z_\beta \nu(f)$ (respectively, $Z_\infty \nu(f)$). Given Lemma 2.1, this will be accomplished if we can prove that the distribution of $X_t(f)/X_t(1)$ under $\mathbb{P}^\mu_{\beta t}$ (respectively, $\mathbb{P}^\mu_\infty$) converges weakly as $t \to \infty$ to the unit point mass at $\nu(f)$.

Consider first of all part (i) with $\beta = 1$. Given $\epsilon, \delta > 0$ we have

$$\mathbb{P}^\mu_t(|X_t(f)/X_t(1) - \nu(f)| > \epsilon) \le \mathbb{P}^\mu_t(|X_t(f) - \nu(f)X_t(1)| > \delta t\epsilon) + \mathbb{P}^\mu_t(X_t(1) \le \delta t).$$

Observe that

$$\mathbb{P}^\mu_t(|X_t(f) - \nu(f)X_t(1)| > \delta t\epsilon) \le \delta^{-2}\epsilon^{-2}t^{-2}[1 - \exp(-\mu(1)t^{-1})]^{-1} \times \mathbb{P}^\mu([X_t(f) - \nu(f)X_t(1))]^2);$$

and so, from Lemmas 2.3 and 2.1,

$$\limsup_{t\to\infty} \mathbb{P}^\mu_t(|X_t(f)/X_t(1) - \nu(f)| > \epsilon) \le \int_0^\delta e^{-x}dx.$$

As δ is arbitrary, the result follows in this case.

Now consider part (ii). Given $\epsilon, \delta > 0$ we have

$$\mathbb{P}^\mu_\infty(|X_t(f)/X_t(1) - \nu(f)| > \epsilon) \le \mathbb{P}^\mu_\infty(|X_t(f)/X_t(1) - \nu(f)|X_t(1)^{1/2} > \delta^{1/2}t^{1/2}\epsilon) + \mathbb{P}^\mu_\infty(X_t(1) \le \delta t).$$

Observe that

$$\mathbb{P}^{\mu}_{\infty}(|X_t(f)/X_t(1) - \nu(f)|X_t(1)^{1/2} > \delta^{1/2}t^{1/2}\epsilon) \leq \delta^{-1}\epsilon^{-2}t^{-1}\mu(1)^{-1} \times \mathbb{P}^{\mu}([X_t(f) - \nu(f)X_t(1)]^2);$$

and so, from Lemmas 2.3 and 2.1,

$$\limsup_{t\to\infty} \mathbb{P}^{\mu}_{\infty}(|X_t(f)/X_t(1) - \nu(f)| > \epsilon) \leq \int_0^{\delta} xe^{-x}dx.$$

The desired result again follows.

Finally, consider part (i) with $\beta \in]1,\infty[$. Given what we have shown in the previous paragraph, the result is immediate from Lemma 2.2.

3. Some examples

EXAMPLE 3.1. Suppose that E is discrete (and hence countable). If there exists a probability measure ν on E such that $\lim_{t\to\infty} P_t(x,\{y\}) = \nu(\{y\})$ for all $x,y \in E$ then Hypothesis (I) holds.

EXAMPLE 3.2. Suppose that E is, in fact, compact and is also a topological Abelian group. Suppose further that ξ is a Lévy process on E, so that the measures $(P_t(0,\cdot))_{t\geq 0}$ form a continuous convolution semigroup. If we let $\hat{E}$ denote the dual of E and write $\langle\cdot,\cdot\rangle$ for the canonical pairing between E and $\hat{E}$, then it is well-known that there exists a function $\phi: \hat{E} \to \mathbb{C}$ such that $\int P_t(0,dy)\langle y,\chi\rangle = \exp(-t\phi(\chi))$ for all $t \geq 0$ and $\chi \in \hat{E}$ (see, for example, Ch. IV of [8]).

Assume that $\Re e\phi(\chi) > 0$ when $\chi \neq 0_{\hat{E}}$. Then $\lim_{t\to\infty} \int P_t(x,dy)\langle y,\chi\rangle = 0$ for $\chi \neq 0_{\hat{E}}$ and so $P_t(x,\cdot)$ converges weakly to ν as $t \to \infty$, where ν is normalized Haar measure. It is possible to metrise E with a translation invariant metric (for instance, we can take any metric and then average over Haar measure). Let $d(\cdot,\cdot)$ be such a metric. Note that if $f: E \to \mathbb{R}$ is continuous then

$$\begin{aligned}|P_tf(x) - P_tf(y)| &= \left|\int P_t(0,dz)\,[f(z-x) - f(z-y)]\right| \\ &\leq \sup\{|f(v) - f(w)| : d(v,w) = d(x,y)\}\end{aligned}$$

and so the family of functions $\{P_tf\}_{t\geq 0}$ is equicontinuous. Applying the Arzéla-Ascoli Theorem, we see that $P_tf \to \nu(f)$ uniformly as $t \to \infty$ and hence Hypothesis (I) holds.

EXAMPLE 3.3. Suppose that $E = \mathbb{R}$ and ξ is a regular diffusion in natural scale. If ξ has finite speed measure m, then from Theorems V.50.11 and V.54.5 of [10] we see that Hypothesis (II) holds with $\nu = m(\cdot)/m(1)$.

EXAMPLE 3.4. Suppose that $E = \mathbb{R}^d$. Suppose that $a^{ij} : \mathbb{R}^d \to \mathbb{R}$, $1 \le i,j \le d$, and $b^j : \mathbb{R}^d \to \mathbb{R}$, $1 \le j \le d$ are in $C^\infty(\mathbb{R}^d)$ with bounded derivatives of all positive orders and that the matrix $(a^{ij}(x))$ is invertible and positive definite for all x. Then there is a unique Feller semigroup (P_t) with infinitesimal generator extending the differential operator $\frac{1}{2}\sum_{ij} a^{ij}\partial_i\partial_j + \sum_j b^j\partial_j$. Moreover, for all $x \in \mathbb{R}^d$ and $t > 0$ we have that $P_t(x,\cdot)$ is absolutely continuous with respect to Lebesgue measure.

Assume that there exists a smooth function p such that $p > 0$, $\int p(x)dx = 1$ and $0 = \frac{1}{2}\sum_{i,j}\partial_i\partial_j(a^{ij}(x)p(x)) - \sum_j \partial_j(b^j(x)p(x))$. Then the argument given in the proof of Theorem 1.6 in [6] shows that Hypothesis (II) holds with $\nu(dx) = p(x)dx$.

REFERENCES

1. K. B. Arthreya and P. E. Ney, *Branching Processes*, Springer, 1972.
2. E. B. Dynkin, *Regular transition functions and regular superprocesses*, Trans. Am. Math. Soc., to appear.
3. N. El Karoui and S. Roelly-Coppoletta, *Study of a general class of measure-valued branching processes; a Lévy–Hinčin representation*, preprint.
4. S. N. Evans and E. Perkins, *Absolute continuity results for superprocesses with some applications*, Trans. Am. Math. Soc., to appear.
5. P. J. Fitzsimmons, *Construction and regularity of measure-valued Markov branching processes*, Isr. J. Math. **64** (1988), 337–361.
6. I. Herbst and L. Pitt, *Diffusion equation techniques in stochastic monotonicity and positive correlations*, Probab. Theory Relat. Fields, to appear.
7. O. Kallenberg, *Random Measures* (3rd edn.), Akademie-Verlag, Academic Press, 1983.
8. K. R. Parthasarathy, *Probability Measures on Metric Spaces*, Academic Press, 1967.
9. S. Roelly-Coppoletta and A. Rouault, *Processus de Dawson–Watanabe conditionné par le futur lointain*, C.R. Acad. Sci. Paris, Série I, **309** (1989), 867–872.
10. L. C. G. Rogers and D. Williams, *Diffusions, Markov Processes and Martingales*, Volume 2: *Itô Calculus*, Wiley, 1987.
11. M. J. Sharpe, *General Theory of Markov Processes*, Academic Press, 1988.
12. S. Watanabe, *A limit theorem of branching processes and continuous state branching processes*, J. Math. Kyoto Univ. **8** (1968), 141–167.